Two-scale decomposition for metastable dynamics in continuous and discrete setting.

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Metastable dynamics is characterized by the existence of at least two time scales on which the system shows different behavior. On the short time-scale, the system will reach some local equilibrium which is confined to some strict subset of the state space. Convergence to the equilibrium of the systems happens on the longest time scales which is characterized by rare transitions between these metastable states.

In the first part based on [5] and [6], we consider a diffusion on an energy landscape in the regime of low temperature which provides the metastable parameter. Metastability is manifested in the so-called Eyring-Kramers formula for the optimal constant in the Poincaré inequality (PI). The proof is based on a refinement of the two-scale approach introduced in [3] and of the mean-difference estimate introduced in [2]. In addition, this approach generalizes to the optimal constant in the logarithmic Sobolev inequality (LSI) for which potential theory [1] and semiclassical analysis [4] are not directly applicable.

The two main ingredients are good local PI and LSI constants, and sharp control of the mean difference between metastable regions. The first is obtained by a construction of Lyapunov function for the diffusion restricted to metastable regions. This mimics the fast convergence of the diffusion to metastable states. The mean-difference is estimated by a transport representation of weighted negative Sobolev-norms. It contains the main contribution to the PI and LSI constant, resulting from exponentially long waiting times of jumps between metastable states of the diffusion.

The last part [7] consists of an outlook of how these ideas carry over to reversible Markov chains. We use the link to the potential theoretic approach and consider metastable Markov chains in the sense of [1]. The metastable parameter could also be, besides temperature, the system size. This provides a two scale decomposition into metastable states. The main ingredient in this setting is the use of negative Sobolev norms in discrete setting together with its variational characterization and its relation to capacities.

References

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