Graduate Seminar

PDEs as Gradient Flows

WS 2012/2013

André Schlichting (schlichting@iam.uni-bonn.de)

Topics

(1) Porous medium equation (PME) as gradient flow (GF)
  a) Overview: Classic theory and properties of PME (mass-conservation, finite speed of propagation for $m > 1$, Barenblatt solutions)
  References: [Ott01, Section 1.1], [Vaz06, Chapter 1 and 4].
  b) Abstract GF and two GF interpretations of the PME
  References: [Ott01, Section 1.2 and 1.3], [Lei08, Section 1–3], [OW05, Section 1–2]
  c) Physical background, derivation and physical interpretation of entropic GF of PME
  References: [Vaz06, Chapter 2], [Ott01, Section 2], [Lei08, Section 4]
  d) Mathematical properties (self-similarity, asymptotic results) via GF interpretation
  References: [Ott01, Section 3], [Lei08, Section 11–14]
  e) Towards rigorous results (only smooth case)
  References: [Ott01, Section 5.3, Proposition 1]

(2) Time-discretization of PDEs via gradient flow interpretation
  a) Time-discretization of porous medium equation and Fokker-Planck equations
  References: [Ott01, Section 4.6], [JKO98, Section 1-4]
  b) Convergence of time-discrete scheme for Fokker-Planck equation
  References: [JKO98, Section 5]

(3) "Otto Calculus": Riemannian structure and abstract gradient flows
  (a) Riemannian structure of entropic GF (isometric submersion, induced distance)
  References: [Ott01, Section 4.1–4.3], [Lei08, Section 5–9]
  (b) Contractive properties and convexity: Eulerian point of view
  References: [OW05, Section 3–4]
  (c) Relation to functional inequalities
  References: [OV00], [Vil03, Chapter 9]
  (d) "Otto Calculus"
  References: [Vil09, Chapter 15]

(4) Recent developments
  a) Beyond scalar conservation laws
  References: [Mie11]
  b) Beyond entropic gradient flows
  References: [DNS09]
  c) Slow motions of gradient flows
  References: [OR07]
  d) Time-discretization of gradient flow with dynamic constraint
  References: [HNV11]

Schedule

10.10.12) * Distribution of topics, introduction and Riemannian formulation
17.10.12) * Basics: Wasserstein, Conservation laws, Benamou-Brenier, Fokker-Planck equation
24.10.12) I Classic porous medium, abstract GF and GF for PME [1a, 1b]
31.10.12) II Time-discretization Fokker-Planck and PME [2a]
07.11.12) III Physical/Mathematical argument GF for PME [1c, 1d]
14.11.12) IV Riemannian structure I (submersion) [3a]
21.11.12) I Riemannian structure II (induced metric) and Wasserstein distance [3a]
28.11.12) II Convergence of time-discretization [2b]
05.12.12) III Rigorous mathematical results for PME [1e]
12.12.12) IV Convexity and contraction from Eulerian point of view [3b]
19.12.12) * Functional inequalities [3c]
09.01.13) I recent developments [4a]
16.01.13) II recent developments [4d]
23.01.13) III recent developments [4c]
30.01.13) IV recent developments [4b]

References

[DNS09] J. Dolbeault, B. Nazaret, and G. Savaré. "A new class of transport distances between measures". *Calculus of Variations and Partial Differential Equations* 34.2 (June 2009), pp. 193–231.

[HNV11] M. Herrmann, B. Niethammer, and J. J. L. Velázquez. "Kramers and non-Kramers Phase Transitions in Many-Particle Systems with Dynamical Constraint" (Oct. 2011), p. 33.

[JKO98] R. Jordan, D. Kinderlehrer, and F. Otto. "The Variational Formulation of the Fokker-Planck Equation". *SIAM Journal on Mathematical Analysis* 29.1 (1998).

[Lei08] H. K. Lei. *Some Notes on: The Geometry of Dissipative Evolution Equations: The Porous Medium Equation by Felix Otto*. 2008.

[Mie11] A. Mielke. "A gradient structure for reaction–diffusion systems and for energy-drift-diffusion systems". *Nonlinearity* 24.4 (Apr. 2011), pp. 1329–1346.

[OR07] F. Otto and M. Reznikoff. "Slow motion of gradient flows". *Journal of Differential Equations* 237.2 (June 2007), pp. 372–420.

[Ott01] F. Otto. "The Geometry of Dissipative Evolution Equations: the Porous Medium Equation". *Communications in Partial Differential Equations* 26.1&2 (Jan. 2001), pp. 101–174.

[OV00] F. Otto and C. Villani. "Generalization of an Inequality by Talagrand and Links with the Logarithmic Sobolev Inequality". *Journal of Functional Analysis* 173.2 (June 2000), pp. 361–400.

[OW05] F. Otto and M. Westdickenberg. "Eulerian Calculus for the Contraction in the Wasserstein Distance". *SIAM Journal on Mathematical Analysis* 37.4 (2005), p. 1227.

[Vaz06] J. L. Vazquez. *The Porous Medium Equation*. Oxford University Press, Oct. 2006.

[Vil03] C. Villani. *Topics in Optimal Transportation*. Graduate S. American Mathematical Society, 2003.

[Vil09] C. Villani. *Optimal Transport*. Ed. by C. Villani. Vol. 338. Grundlehren der mathematischen Wissenschaften. Berlin, Heidelberg: Springer-Verlag, 2009.