Lecture
Stochastic Control Theory (MSc.).

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Stochastic control theory deals with the optimization of systems, which stochastic processes can represent. Thereby it is possible to influence the stochastic process by an external control to achieve an optimization goal. This includes, for example:

(i) Optimal stopping: The freezing of the system by stopping the stochastic process. A typical example in financial market models is whether and when a holder of an American put option should optimally exercise it.

(ii) Classical control: The control of the system is based on the change of the local drift and/or the local volatility of the stochastic process. For this purpose, one can imagine the optimization problem in which a lake is to be optimally fished.

(iii) Singular control: The system can be influenced by reflecting the stochastic process at chosen bounds. This optimization problem occurs in controlling rockets whose trajectory needs to be kept in a certain corridor with as little fuel input as possible.

(iv) Impulse control: The control of the system by impulses that cause the system to jump. This control is very similar to the classical control, with the difference that the solution's trajectories are no longer continuous. As an example serves the forestry, which seeks optimal strategies for the sustainable management of the forest.

The lecture will start by explaining the necessary results from stochastic analysis and stochastic processes. Afterward, we will deal with classical stochastic control. In particular, we will discuss the connection to martingale theory and partial differential equations. This will allow us to solve the classical control problem. We will see that the theory developed so far requires a weaker solution term for the underlying partial differential equation. This is the motivation to introduce viscosity solutions to approximate the control problem's value function in general. In the last part of the lecture, we will consider the control of many interacting agents that leads to mean-field games.

The lecture is intended for master students who want to specialize in the field of analysis and stochastics. A background on partial differential equations or stochastic analysis is useful.